Problem name: Strategy

Language: English

Solution

We can easily calculate the k-th element by calculating each element in the sequence, however such solution is worth only 20 points. In order to do it fast, we need to notice that some parts of the sequence are following a certain pattern. Using that observation we can skip large parts of the sequence at once.

The first step is to calculate consecutive elements until we obtain three elements in non-increasing order. Then, as a second step, we observe a repeating pattern. For example:

39, 35, 4, 31, 27, 4, 23, 19, 4, 15, 11, 4, 7, 3, 4

If the first two elements are A and B (A >= B >= A-B), the sequence is divided into three element blocks, k-th block having the form

A-k*2d, B-k*2d, d,

where d = A-B.

We have B/(2d) blocks that follow this pattern. If the k-th element belongs to one of these blocks, we find its value directly by calculating the block to which it belongs, its position inside that block, and then applying the former rule. If the k-th element is occurring after all these blocks, we restart the process by going to the first step again.

It should be easy to intuitively conclude that this algorithm is fast. However, the precise proof that it takes at most logarithmic time (as the function of the first number) is a bit technical.